When L is expanded as a series in powers of $B(3S^*)^{1/3}$, it is easily shown that the solutions for the shear profile also apply to L, that is,

$$L = 1 - F^{(0)} - G^{(1)}B(3S^*)^{1/3} \tag{13}$$

When the enthalpy in the body boundary layer is assumed to be given by the Crocco integral relation $H - H = (H - H_w)(1 - u/u)$, then $W(u/u, 0) = 1 - u/u$ As shown in Ref 2, for these initial conditions $L = 1 - u/u - W$, thus reducing the number of auxiliary functions from two to one In this case the series solution for W gives

$$W = F^{(0)} - (\xi - G^{(1)}B)(3S^*)^{1/3} \tag{14}$$

The series solutions developed here are in terms of the Crocco coordinate system $[F(u/u, S), H(u/u, S)]$, but, since the location of the dividing streamline also is calculated, the normal distance coordinate Y can be evaluated by means of the integral

$$Y = \int_{u_D/u_e}^{u/u_e} \frac{1}{F}\, d\left(\frac{u}{u}\right)$$

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Experimental Detonation Velocities and Induction Distances in Hydrogen-Air Mixtures

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Detonation velocities and induction distances have been measured for hydrogen-air mixtures at 1 and 2 atm initial pressure in a 74-mm-diam, 54-m long tube The detonation velocities increase with increasing fuel concentration and with increasing initial pressure The detonation induction distances, 630–730 cm, are quite long compared to those of hydrogen-oxygen mixtures; these distances decrease at the higher initial pressure

Introduction

EXPERIMENTAL detonation velocities and induction distances have been measured for various fuel concentrations of hydrogen in air at initial pressures of 1 and 2 atm at ambient initial temperature These studies are a portion of the basic investigation of detonation phenomena which is being conducted in this laboratory Recent results have been presented in Refs 1–10

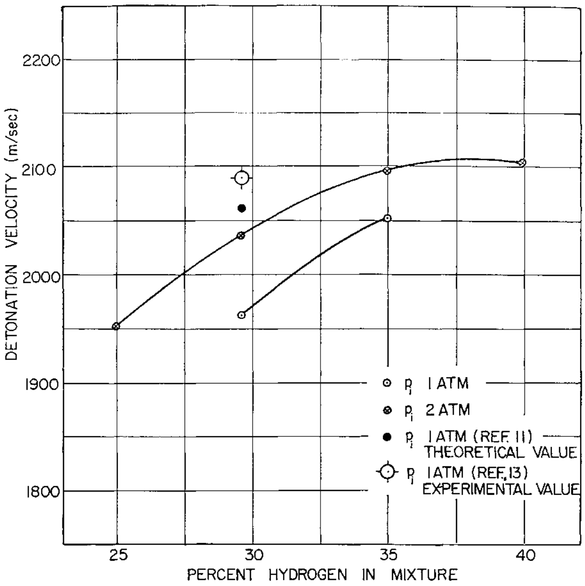

Fig 1 Experimental detonation velocities of hydrogen-air mixtures as a function of fuel concentration

These experiments were conducted in a 74-mm-i d inconel detonation tube, 54 m in length Ionization-type probes were utilized to sense the passage of the flame front Multichannel measurements of the time intervals were made with a 1-Mc chronograph system The probes have a nominal separation of 60 cm along the entire tube length Pyrofuze wire, of 0 005-in diam, was used to ignite the mixtures

A number of experiments were carried out for the same initial conditions of pressure, temperature, and fuel concentration because of the nonrepeatable nature of the flame front in the initiation region These data were averaged and graphed, including the maximum and minimum values, as a function of distance from the ignitor When the flame propagation rates became steady, the detonation velocities were determined Induction distances, the distance from the ignitor to the location in the detonation tube where the flame propagation rate first attains the detonation velocity, were ascertained from these graphs too As with previous data, two detonation induction distances are given; one distance is based on the maximum flame propagation rates and the other on average values

Discussion of Results

The velocities of the stable detonations were determined at various probe positions between 35 and 50 m from the ignitor At this distance, the detonation wave is completely stabilized for hydrogen-air mixtures as determined experimentally from measurements of the flame propagation rates along the entire length of the tube

From Fig 1 it can be seen that the detonation velocity appears to reach a maximum value near 40% fuel concentration for an initial pressure of 2 atm Data obtained at 1 atm follow the same trend as do those measured at 2 atm, but the values are somewhat lower The difference between the detonation velocities for the two values of initial pressure decreases near the maxima

It was not possible to measure detonation velocities for fuel concentrations farther from the stoichiometric value than those tabulated without making a major change in existing instrumentation The nitrogen content in the fuel-oxidizer mixture greatly decreases the flame temperature and, therefore, the degree of ionization of the product gases upon which property the present detection technique depends Special one-stage transistorized amplifiers were designed and fabricated to amplify the meager signals; without the use of these amplifiers little of the present data could have been obtained

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Table 1 Theoretical and experimental values of detonation velocities of hydrogen-air mixtures from Ref 11[a]

Mixture	Detonation velocity, m/sec Theoret	Detonation velocity, m/sec Exptl
$2 H_2 + O_2$	2806	2819
$2 H_2 + O_2 + 1 N_2$	2378	2407
$2 H_2 + O_2 + 3 N_2$	2033	2055
$2 H_2 + O_2 + 5 N_2$	1850	1822

[a] p_i = ambient atmospheric pressure

Some theoretical and experimental values of detonation velocities are reported in Ref 11 for a stoichiometric mixture of hydrogen and oxygen diluted by varying quantities of nitrogen These data are given in Table 1 The theoretical values were graphed, and the detonation velocity for a stoichiometric hydrogen-air mixture (29 6% fuel) was determined as shown in Fig 1 The theoretical value is higher than the experimental value by 100 m/sec

One reason for this difference, besides nonideal conditions, comes from the technique employed to calculate the theoretical values Lewis and Friauf considered only the following equilibria in their calculations

$$2H_2O \rightleftharpoons 2H_2 + O_2$$

$$2H_2O \rightleftharpoons H_2 + 2OH$$

$$H_2 \rightleftharpoons 2H$$

Since nitric oxide, atomic oxygen, and atomic nitrogen are present in the hot gas mixture, it is believed that this limitation in the calculation technique is one of the reasons for the difference between the experimental and theoretical values Also, improved thermodynamic data have become available since their calculations were made Their value of detonation velocity for a stoichiometric mixture of hydrogen and oxygen differs too, probably for the same reasons, from values calculated more recently (e g , Ref 2)

The experimental values reported in Ref 11 came from Ref 12; the original data were given in Ref 13 Dixon conducted his experiments in a 9-mm-i d lead pipe, both coiled and straight, 100 m long For comparison purposes, the interpolated experimental detonation velocity of Dixon for a stoichiometric hydrogen-air mixture is shown in Fig 1 This value is appreciably higher than that obtained for a similar mixture in this investigation The difference could arise from several factors Dixon's tube was 9 mm in diameter, whereas the present one is 74 mm From other studies it is well known that the detonation velocity decreases as the tube diameter is increased Also, there is a good possibility that the chronograph employed in the older studies was relatively inaccurate compared to the current type Even so, the difference is only some 6%

No other data were found in the literature for detonation induction distances in hydrogen-air mixtures The experimental values, Table 2, are similar regardless of whether the maximum or average flame propagation rates were employed in the determination The flame propagation rates rose rather sharply to the detonation value in the induction zone as exemplified by the data presented in Fig 2 for the stoichiometric mixture at 2-atm initial pressure Figure 3 illustrates the variation in flame propagation rates along the length of the detonation tube for the stoichiometric mixture at 1-atm initial pressure

Table 2 Detonation velocities and induction distances of hydrogen-air mixtures[a]

p_i, atm	Mole % fuel	Detonation velocity, m/sec	Induction distance, cm A	Induction distance, cm B
1	29 6	1963	730	740
1	35	2053		
2	25	1953	700	710
2	29 6	2037	630	645
2	35	2096	640	670
2	40	2105		

[a] Initial temperature = ambient value A = based on maximum flame propagation rates B = based on average flame propagation rates

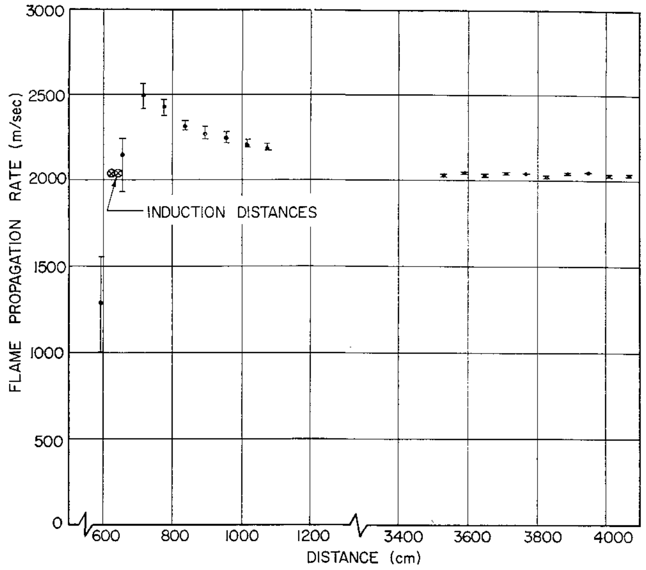

Fig 2 Rate of flame propagation in a hydrogen-air mixture; mole % H_2 = 29 6, p_i = 2 atm, t_i = ambient, Pyrofuze ignitor

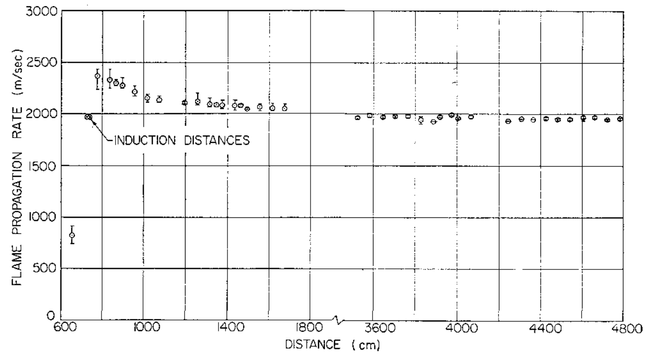

Fig 3 Rate of flame propagation in a hydrogen-air mixture; mole % H_2 = 29 6, p_i = 1 atm, t_i = ambient, Pyrofuze ignitor

As with other combustible mixtures[1–10] the detonation induction distance of hydrogen-air mixtures decreases with an increase in initial pressure Also, the minimum induction distance appears to occur near the stoichiometric mixture at 2-atm initial pressure which can be seen by making a graph of the data in Table 2 These results are similar to those determined experimentally for other fuel-oxidizer mixtures As anticipated, the induction distances of hydrogen-air mixtures are appreciably longer than those of hydrogen-oxygen mixtures[3] by a factor of approximately 10 for equal equivalence ratios

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Laminar Boundary Layer with Fluid Injection

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This paper examines the distribution of properties through the boundary layer, and in particular the temperature, instead of heat transfer at the wall and skin friction which are the usual objects of interest The simplifying assumption of Prandtl, Lewis, and Schmidt numbers of 1 yields a solution with interesting results pertaining to boundary-layer cooling Cooling through the boundary layer is reduced to a relatively simple function of mass of fluid coolant entering the layer and its specific heat An injectant with a very high specific heat can lower the maximum temperature to about $\frac{1}{4}$ its normal level for air

Nomenclature

x,y = geometric coordinates—distance along and normal to surface, respectively
p,T = thermodynamic properties of pressure, temperature
ρ = density
h = enthalpy
u = velocity in x direction
v = velocity in y direction
c = mass fraction
C_p = specific heat at constant pressure
μ = viscosity
k = thermal conductivity
Pr = Prandtl number $\mu Cp/k$
Re = Reynolds number $\rho ux/\mu$
Le = Lewis number $CpD\rho/k$
D = binary diffusion coefficient
δ = thickness of boundary layer
Sc = Schmidt number $\mu/\rho D = (Pr/Le)$
$\dot{m}$ = mass flow per unit area
C_f = local coefficient of friction $\tau_w/\frac{1}{2}\rho_\infty u_\infty{}^2$
τ_w = shear at the wall $\tau_w = \mu_w(\partial u/\partial y)_w$
$C_1{}^*$ = C_1 for Cp^2T/ho_∞ a maximum
C' = Chapman and Rubesin constant from $\mu/\mu_\infty = C'T/T_\infty$

Subscripts

w = wall condition
∞ = freestream outside of boundary layer
0 = total condition
1 = injected substance
2 = original substance in boundary layer
i = ith species, incompressible

EXTENSIVE uses of ablation to protect bodies in high-speed flight have focused attention on the effect on the boundary layer of mass injection More recently the problem of radiation has created an interest in quenching the high temperatures in a boundary layer by this mass injection

In examining the laminar boundary layer equations for a mixture as given, for example, in Ref 1 for the assumptions of Prandtl and Lewis numbers = 1, and without pressure gradient or chemical reaction the continuity for a single species, momentum and energy equations, respectively, can be written as follows:

$$\rho u\frac{\partial C_i}{\partial x} + \rho v\frac{\partial C_i}{\partial y} = \frac{\partial}{\partial y}\left(\mu\frac{\partial C_i}{\partial y}\right) \tag{1}$$

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{2}$$

$$\rho u\frac{\partial h_0}{\partial x} + \rho v\frac{\partial h_0}{\partial y} = \frac{\partial}{\partial y}\left(\mu\frac{\partial h_0}{\partial y}\right) \tag{3}$$

Many authors have indicated the solution to Eq (3) can be written in terms of the solution to Eq (2) as follows:

$$h + u^2/2 = (h_{0\infty} - h_w)u/u_\infty + h_\infty \tag{4}$$

with the boundary conditions

$$y = 0 \qquad u = 0 \qquad h = h_w \qquad C_1 = C_{1w}$$
$$y = \delta \qquad u = u_\infty \qquad h = h_\infty \qquad C_1 = 0$$

Likewise, the relation between C and u can be obtained

$$C_1/C_{1w} = 1 - u/u_\infty \tag{5}$$

Eliminating u/u_∞ between Eqs (4) and (5) provides the corresponding relation between h and C,

$$h = h_w(C_1/C_{1w}) + (1 - C_1/C_{1w})(h_\infty + C_1/C_{1w}u_w{}^2/2) \tag{6}$$

Differentiating either Eq (4) or (6) yields h maximum

$$\frac{h_{\max}}{h_{0\,\infty}} = \frac{1}{4}\left[1 + 2\left(\frac{h_w}{h_{0\,\infty}}\right) + \left(\frac{h_w}{h_{0\,\infty}}\right)^2\right] \text{ for } \frac{h_\infty}{h_{0\,\infty}} \ll 1 \tag{7}$$

For a cold wall $h_w/h_{0_\infty} \ll 1$, Eq (7) further reduces to $h_{\max}/h_{0_\infty} \simeq \frac{1}{4}$ as has been noted in Ref 1, but it is noted here that the result is independent of both velocity and concentration profiles as well as the amount and substance being injected The temperature, however, does depend on both the con-

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